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Current–voltage characteristic of quantum wires

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Abstract. The current–voltage characteristic (CVC) of quantum wires at low temperatures is considered theoretically. Three mechanisms of electron relaxation are taken into account: (i) elastic momentum relaxation by impurities or defects, (ii) inelastic energy relaxation by acoustic phonons and (iii) optical phonon emission. The latter causes a very fast energy loss for electrons reaching the energy of optical phonons. If the two first mechanisms are characterized by energy-independent relaxation times, the CVC has a sublinear shape with saturation. If the elastic relaxation time increases with electron energy, which is typical for impurity scattering, the CVC is characterized by a non-monotonic field dependence of differential resistance with a minimum at some critical field. Most analytical results are obtained for a wire with one occupied subband but the main features of a multi-subband case are also discussed.

1. Introduction

In the past few years, the physics of low-dimensional electron systems, so far concerned almost entirely with two-dimensional structures, has had great success in studying one-dimensional electron systems—quantum wires. These systems demonstrate a series of new interesting phenomena, in particular, in their kinetic properties. The physical nature of these phenomena depends drastically on the relation between the wire length and the electron mean free path. For short, ballistic wires, the current is determined by emission from contacts and described by the well known Landauer formula [1] which can be generalized to the case of a high applied voltage [2] and describe the current–voltage characteristic (CVC) of the system.

In the present work the opposite limit of long wires will be discussed. In this case, for not very low temperatures, when the localization phenomena are of minor importance, the CVC can be found from the kinetic equation and is determined by mechanisms of electron scattering. The latter in quantum wires have a series of specific features such as, for example, the absence of low-angle scattering [3] and the suppression of electron–electron scattering [4]. The electron–phonon interaction in one-dimensional systems also has some specific features (see, e.g., [5] and references therein). As a result, the distribution function of one-dimensional electrons in an applied electric field and, hence, CVC may differ drastically from those in three- and two-dimensional systems. In spite of a large number of papers on momentum and energy relaxation in one-dimensional electron systems, very few of them discuss directly the problem of CVC. There are several numerical calculations of the drift velocity–electric field dependence [6–9] but, as with any numerical results, they do not give

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us the possibility of tracing the dependences of CVC on different parameters characterizing the sample and scattering processes in it.

Some analytical results were obtained in [10]. The author demonstrated that the predominance of optical phonon scattering in the absence of electron–electron interactions might cause an anomalous carrier cooling and a 'jagged' shaped distribution function. One may expect that an additional quasi-elastic scattering (say, by acoustic phonons) will smear the distribution function singularities and influence the CVC. This problem was mentioned by the author very briefly.

In the present paper we consider a simple model which takes into account the optical phonon scattering as well as additional elastic and quasi-elastic scattering in quantum wires. The model allows us to obtain analytical results for the distribution function in an electric field and for the CVC in quantum wires.

2. Basic equations

We consider a quantum wire with the standard quasi-one-dimensional dispersion law:

$$E = E_N + \frac{p^2}{2m} \tag{1}$$

where p is the momentum along the wire (x-axis) and N numerates discrete energy levels for the motion in yz-plane. Three main scattering mechanisms will be considered:

(i) optical phonon emission,

(ii) elastic impurity or defect scattering and

(iii) quasi-elastic acoustic phonon scattering providing energy relaxation for electrons with energies less than the optical phonon energy $\hbar\Omega$.

The characteristic times of these processes will be assumed to form the hierarchy $\tau_{op} \ll \tau_i \ll \tau_{ac}$, which is typically the case in most semiconductors. The lattice temperature will be assumed low enough that processes of optical phonon absorption can be ignored.

We begin with the purely one-dimensional case with one occupied subband and describe electrons by their distribution function $f^{\pm}(p)$ where superscripts $+$ and $-$ correspond to positive and negative momentum direction (we have omitted the subband index). In the momentum interval $|p| < p_0 \equiv \sqrt{2m\hbar\Omega}$ the optical phonon emission is prohibited by energy conservation. Since the elastic impurity scattering may cause only transitions between p and $-p$ states, the kinetic equation for $|p| < p_0$ has the following form:

$$eF\frac{\mathrm{d}f^-(p)}{\mathrm{d}p} = \frac{f^+(-p) - f^-(p)}{\tau_i(p)} - \frac{f^-(p) - f_0(p)}{\tau_{ac}(p)} \tag{2}$$

$$eF\frac{\mathrm{d}f^+(p)}{\mathrm{d}p} = \frac{f^-(-p) - f^+(p)}{\tau_i(p)} - \frac{f^+(p) - f_0(p)}{\tau_{ac}(p)}. \tag{3}$$

Here F is the electric field and f_0 is the equilibrium Fermi distribution function.

In the case $\tau_{op} \ll \tau_i, \tau_{ac}$ we may assume that, when the electron momentum reaches p_0, an optical phonon is emitted immediately causing electron transition from the $p = p_0$ to the $p = 0$ state†. The continuity of a flux in p-space at $p = 0$ gives the following condition:

$$f^+(0) = f^-(0) + f^+(p_0) \tag{4}$$

† Since the electric field accelerates electrons in the positive direction of the x-axis, then in the case when both thermal and Fermi energies are less than $\hbar\Omega$, the state $p = -p_0$ can never be reached.

which can be used as a boundary condition for the system (2), (3). The other condition to be used is the normalizing condition

$$\int_{-\infty}^{0} f^{-}(p)\,\mathrm{d}p + \int_{0}^{\infty} f^{+}(p)\,\mathrm{d}p = \int_{-\infty}^{\infty} f_0(p)\,\mathrm{d}p = \pi\hbar n \tag{5}$$

reflecting the conservation of one-dimensional electron density n.

For further calculations it is more convenient to replace f^- and f^+ with the symmetric $f_s = (f^+ + f^-)/2$ and antisymmetric $f_a = (f^+ - f^-)/2$ components of the distribution function. In terms of these quantities the kinetic equations (2) and (3) for $\tau_i \ll \tau_{ac}$ can be written as

$$eF\frac{\mathrm{d}f_s(p)}{\mathrm{d}p} = -\frac{2f_a(p)}{\tau_i(p)} \tag{6}$$

$$eF\frac{\mathrm{d}f_a(p)}{\mathrm{d}p} = -\frac{f_s(p) - f_0(p)}{\tau_{ac}(p)}. \tag{7}$$

The conditions (4) and (5) are transformed into

$$2f_a(0) = f_s(p_0) + f_a(p_0) \tag{8}$$

$$\int_{0}^{\infty} f_S(p)\,\mathrm{d}p = \int_{0}^{\infty} f_0(p)\,\mathrm{d}p = \frac{\pi\hbar n}{2}. \tag{9}$$

3. Analytical results for constant relaxation times

The equation system (6), (7) can be solved explicitly for the case of momentum-independent relaxation times τ_i and τ_{ac}. If we assume electrons to be completely degenerate and introduce the dimensionless momentum $P = (p/eF)\sqrt{2/(\tau_{ac}\tau_i)}$, then the general solution for f_s can be written as

$$\begin{aligned} &f_s(P) = 1 + A\exp(P - P_F) + B\exp(P_F - P) && \text{for } 0 < P < P_F \\ &f_s(P) = C\exp(P - P_F) + D\exp(P_F - P) && \text{for } P > P_F. \end{aligned} \tag{10}$$

Here P_F is the dimensionless Fermi momentum. Constants A, B, C, D are to be found from (8) and (9) and the matching conditions at $P = P_F$. This gives

$$A + 1/2 = C = \frac{(1-\alpha)[1 - \exp(2P_F)]}{2[\exp(P_0) - 1][(\alpha+1)\exp(P_0) - 1 + \alpha]} \tag{11}$$

$$B + 1/2 = D = \frac{(1+\alpha)\exp(2P_0)[1 - \exp(2P_F)]}{2[\exp(P_0) - 1][(\alpha+1)\exp(P_0) - 1 + \alpha]} \tag{12}$$

where $\alpha = \sqrt{\tau_i/\tau_{ac}}$.

Now we can find the CVC. The current

$$j = \frac{4e}{\pi\hbar m}\int_0^{p_0} p f_a(p)\,\mathrm{d}p = -\frac{e^3F^2\tau_i\sqrt{2\tau_{ac}\tau_i}}{\pi\hbar m}\int_0^{P_0} P\frac{\mathrm{d}f_s}{\mathrm{d}P}\,\mathrm{d}P$$

is easily calculated with the help of (10), (11) and (12) and, returning to ordinary units, we have eventually

$$\begin{aligned} j = \frac{2e^2F\tau_i}{\pi\hbar m}\{&p_F - [p_0\sqrt{\tau_i/\tau_{ac}}\exp(\sqrt{2}p_0/eF\sqrt{\tau_{ac}\tau_i})[\exp(\sqrt{2}p_F/eF\sqrt{\tau_{ac}\tau_i}) \\ &- \exp(-\sqrt{2}p_F/eF\sqrt{\tau_{ac}\tau_i})]][[\exp(\sqrt{2}p_0/eF\sqrt{\tau_{ac}\tau_i}) - 1] \\ &\times[(\sqrt{\tau_i/\tau_{ac}} + 1)\exp(\sqrt{2}p_0/eF\sqrt{\tau_{ac}\tau_i}) + \sqrt{\tau_i/\tau_{ac}} - 1]]^{-1}\}. \end{aligned} \tag{13}$$

The expression (13) has simple asymptotics. At $F \to 0$ the first term dominates and we have the ordinary Ohm law: $j = ne^2\tau_i F/m$. At $F \to \infty$ electrons become distributed uniformly in the momentum interval $0 < p < p_0$ and the current saturates:

$$j = j_s \equiv \frac{enp_0}{\sqrt{2}m} = \text{ const.}(F). \tag{14}$$

The saturation of current at high fields is a consequence of our boundary condition (4) corresponding to the limit $\tau_{op} \to 0$. More rigorous analysis for a finite τ_{op} (see numerical calculations in [6]) shows that instead of saturation, very slow growth of current (quasi-saturation) must take place, followed by its fast growth due to the electron 'runaway' at very high F, so our approach is expected to become inadequate for very large electric fields.

4. Energy-dependent relaxation times

The energy dependence of τ_i and τ_{ac} ignored in the previous section may change noticeably the character of the CVC. Typically, the relaxation times decrease with decrease in energy due to the high density of states near the subband edge in a one-dimensional system. As we saw in the previous section, the answer for the CVC is almost independent of τ_{ac} which allows us not to take into account its energy dependence. In contrast, the dispersion of τ_i can be of primary importance. We consider the model case of a linear relaxation time–momentum dependence: $\tau_i(p) = \tau_0 p$. Such a dependence takes place when the matrix element of scattering is momentum independent (for instance, for the scattering by short-range impurity potential) and $\tau_i(p)$ is determined only by the one-dimensional density of states $\sim(E - E_1)^{-1/2}$ (see, e.g., [11]).

In this case the basic equation system (6), (7) can be written as

$$\tilde{P}\frac{\mathrm{d}^2 f_s}{\mathrm{d}\tilde{P}^2} + \frac{\mathrm{d}f_s}{\mathrm{d}\tilde{P}} = f_s(\tilde{P}) - f_0(\tilde{P}) \tag{15}$$

with $\tilde{P} = 2p/(e^2F^2\tau_0\tau_{ac})$ and for degenerate electrons has the following general solution:

$$\begin{aligned} f_s(\tilde{P}) &= 1 + AI_0(2\sqrt{\tilde{P}}) + BK_0(2\sqrt{\tilde{P}}) \qquad &&\text{for } 0 < \tilde{P} < \tilde{P}_F \\ f_s(\tilde{P}) &= CI_0(2\sqrt{\tilde{P}}) + DK_0(2\sqrt{\tilde{P}}) \qquad &&\text{for } \tilde{P} > \tilde{P}_F \end{aligned} \tag{16}$$

where I_0 and K_0 are the Bessel functions of an imaginary argument.

Applying the same boundary conditions as in section 3, we find the equation system for the constants in (16):

$$\begin{aligned} &AI_0(2\sqrt{\tilde{P}_F}) + BK_0(2\sqrt{\tilde{P}_F}) - CI_0(2\sqrt{\tilde{P}_F}) - DK_0(2\sqrt{\tilde{P}_F}) = -1 \\ &AI_1(2\sqrt{\tilde{P}_F}) - BK_1(2\sqrt{\tilde{P}_F}) - CI_1(2\sqrt{\tilde{P}_F}) - DK_1(2\sqrt{\tilde{P}_F}) = 0 \\ &\frac{eF\tau_0}{2}[B + C\sqrt{\tilde{P}_0}I_1(2\sqrt{\tilde{P}_0}) - D\sqrt{\tilde{P}_0}K_1(2\sqrt{\tilde{P}_0})] - CI_0(2\sqrt{\tilde{P}_0}) - DK_0(2\sqrt{\tilde{P}_F}) = 0 \\ &A\sqrt{\tilde{P}_F}I_1(2\sqrt{\tilde{P}_F}) + B[\tfrac{1}{2} - \sqrt{\tilde{P}_F}K_1(2\sqrt{\tilde{P}_F})] + C[\sqrt{\tilde{P}_0}I_1(2\sqrt{\tilde{P}_0}) - \sqrt{\tilde{P}_F}I_1(2\sqrt{\tilde{P}_F})] \\ &\qquad + D[\sqrt{\tilde{P}_F}K_1(2\sqrt{\tilde{P}_F}) - \sqrt{\tilde{P}_0}K_1(2\sqrt{\tilde{P}_0})] = 0. \end{aligned} \tag{17}$$

After calculation of the constants, the current can be readily found:

$$\frac{j}{j_s} = -\frac{\alpha x^5}{8\phi}\left[(A - C)F_1\left(\frac{2\sqrt{\phi}}{x}\right) - (B - D)F_2\left(\frac{2\sqrt{\phi}}{x}\right) + CF_1\left(\frac{2}{x}\right) - DF_2\left(\frac{2}{x}\right)\right]. \tag{18}$$

Here $F_1(y) = \int_0^y t^4 I_1(t)\,\mathrm{d}t$, $F_2(y) = \int_0^y t^4 K_1(t)\,\mathrm{d}t$, $\phi = p_F/p_0$, $\alpha = (\tau_0 p_0/\tau_{ac})^{1/2}$, j_s is given by (14) and the dimensionless electric field $x = eF(\tau_0\tau_{ac}/2p_0)^{1/2}$.

For very low and very high fields analytical expressions for the CVC can be derived. At $x \ll 1$ the CVC is linear: $j = 2\alpha\phi x j_s$ and saturates at $x \gg 1$: $j = \sqrt{2}j_s$. At first glance, the situation is similar to the case $\tau_i =$ constant considered above. However, detailed calculations of the total CVC showed at least two considerable distinctions. First, very fast momentum relaxation at small p inhibits the establishing of uniform distribution of electron momenta at large F which results in very slow, logarithmic saturation of the CVC. Second, the CVC, after the initial linear part, may have an interval of superlinear growth. The effect is connected with the electron heating which increases its mean energy and, hence, increases energy-dependent relaxation time τ_i until the optical phonon emission, resulting in the increase of resistance and saturation of CVC, becomes essential. As a result, the wire resistivity $\rho = F/j$ as well as the differential resistivity $\mathrm{d}F/\mathrm{d}j$ depend on the electric field F non-monotonically. Recent measurements on GaAs/AlGaAs quantum wires whose properties were described in [12] have confirmed the presence of this non-monotonic dependence [13].

The total CVC can be calculated only numerically with the help of (18) and (17). Figure 1 shows the results of such calculations. One can see that the wire resistivity at first decreases and then increases noticeably. This first effect is connected with the electron heating which increases its mean energy $\tilde{E}$ and, hence, energy-dependent relaxation time τ_i. Further increase of the electric field causes effective optical phonon emission resulting in the increase of resistance and saturation of the CVC described in section 3†.

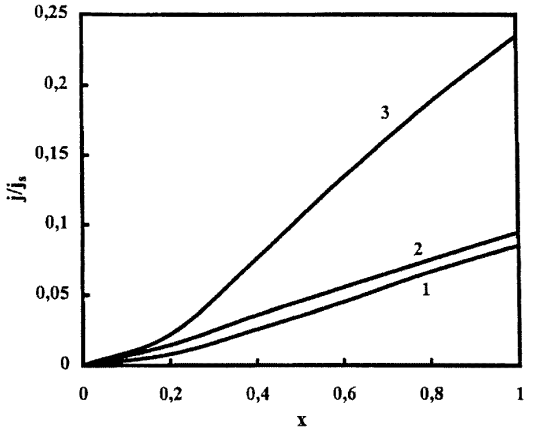
Thus, the wire resistivity–electric field dependence may have a non-monotonic behaviour with a minimum. The minimum becomes less pronounced with the increase of Fermi energy due to decrease of change in $\tilde{E}$ equal to $E_F/2$ at $F \to 0$ and to $\hbar\Omega/2$ at $F \to \infty$. Figure 2 demonstrates the position of this minimum as a function of wire parameters.

It is worth noting that both above-mentioned effects, responsible, respectively, for super- and sublinear CVC, are not specific only for quantum wires and may occur, in principle, in two- and three-dimensional electron systems as well. However, one-dimensional systems are more favourable for observing the non-monotonic behaviour of $\rho(E)$ shown in figure 1. Thanks to the decreasing energy dependence of one-dimensional density of states, τ_i in quantum wires will increase with electron momentum faster than in electron systems of higher dimensionality and the superlinearity of the CVC will be more pronounced. In particular, for the case $\tau_i(p) = \tau_0 p$ considered above, the same scattering mechanism in a two-dimensional electron gas will result in $\tau_i =$ constant(p) which excludes the possibility for the CVC to have a superlinear part and non-monotonic $\rho(E)$.

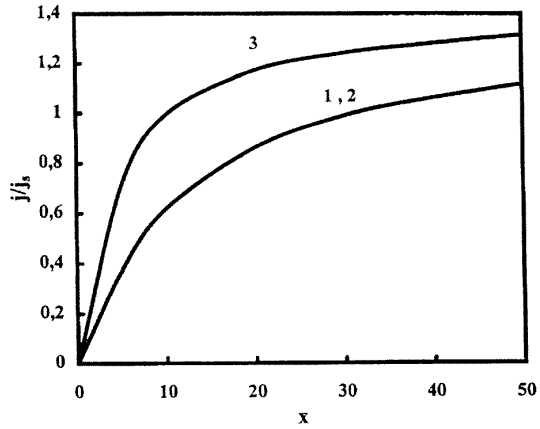
5. Multi-subband case

So far we have considered the purely one-dimensional case with one single occupied subband. Such an approach is correct at $E_F < E_2$ in low electric fields but it may become not the case in higher fields where hot electrons may acquire enough energy to undergo intersubband transitions. Strictly speaking, the formulae of the previous sections are adequate only provided the inequality $\hbar\Omega < \Delta \equiv E_2 - E_1$ takes place as well. Otherwise, we must take into account the contribution of higher subbands with their distribution functions $f_i^{\pm}$. For instance, in the case of two subbands, (2) and (3) must be replaced by the following

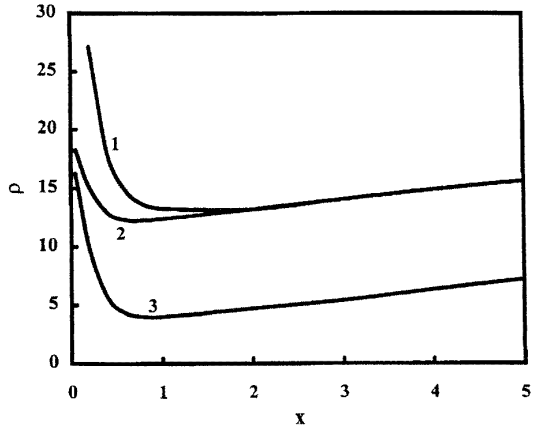
† As already mentioned at the end of section 3, in a more realistic model with non-zero τ_{op} the CVC will have no saturation.



(a)



(b)



(c)

Figure 1. Current–voltage characteristics $j(F)$ at (a) low and (b) high electric fields and (c) resistivity $\rho = F/j$ of a quantum wire for $\alpha = 0.1$, $\phi = 0.1$ (curve 1), $\alpha = 0.1$, $\phi = 0.3$ (curve 2) and $\alpha = 0.3$, $\phi = 0.1$ (curve 3). Here $x = eF(\tau_0\tau_{ac}/2p_0)^{1/2}$ is the dimensionless electric field.

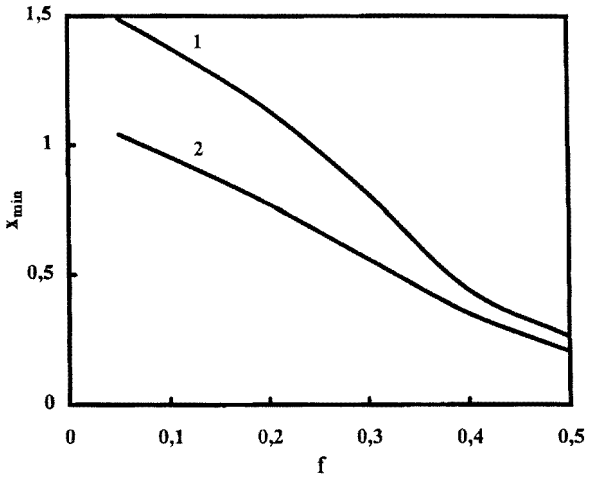


Figure 2. The dependence of dimensionless electric field $x_{min} = eF_{min}(\tau_0\tau_{ac}/2p_0)^{1/2}$ corresponding to the minimum of $\rho = F/j$ on the dimensionless Fermi momentum $\phi = p_F/p_0$ for $\alpha = 0.1$ (curve 1) and $\alpha = 0.3$ (curve 2).

more general system:

$$eF\frac{\mathrm{d}f_1^{\pm}(p)}{\mathrm{d}p} = \frac{f_1^{\mp}(-p) - f_1^{\pm}(p)}{\tau_{11}} + \frac{f_2^{+}(\sqrt{p^2 - 2m\Delta}) - f_1^{\pm}(p)}{\tau_{12}} + \frac{f_2^{-}(-\sqrt{p^2 - 2m\Delta}) - f_1^{\pm}(p)}{\tau_{12}} - \frac{f_1^{\pm}(p) - f_0(p)}{\tau_{ac}} \qquad |p| < p_0 \tag{19}$$

$$eF\frac{\mathrm{d}f_2^{\pm}(p)}{\mathrm{d}p} = \frac{f_2^{\mp}(-p) - f_2^{\pm}(p)}{\tau_{22}} + \frac{f_1^{+}(\sqrt{p^2 + 2m\Delta}) - f_2^{\pm}(p)}{\tau_{12}} + \frac{f_1^{-}(-\sqrt{p^2 + 2m\Delta}) - f_2^{\pm}(p)}{\tau_{12}} - \frac{f_2^{\pm}(p) - f_0(p)}{\tau_{ac}} \qquad |p| < \sqrt{p_0^2 - 2m\Delta}. \tag{20}$$

Here τ_{ij} are elastic relaxation times for both intra- and intersubband scattering (which are, generally speaking, momentum dependent). We do not consider here the particular resonant case $\hbar\Omega = \Delta$ [14].

Electrons in the first subband with $p > p_0$ as well as those in the second subband with $p > \sqrt{p_0^2 - 2m\Delta}$ can fall to the bottom of the first subband by emitting optical phonons. If the emission processes are very fast, then

$$f_1^{+}(0) = f_1^{-}(0) + f_1^{+}(p_0) + f_2^{+}(\sqrt{p_0^2 - 2m\Delta}). \tag{21}$$

In the second subband, the point $p = 0$ has no singularity and

$$f_2^{+}(0) = f_2^{-}(0). \tag{22}$$

Though the system (19)–(22) is too complicated to be solved analytically, some qualitative conclusions can be drawn.

Hot electrons in the second subband can emit optical phonons and pass to the first subband with the characteristic rate τ_{op}^{-1}. The returning process $1 \to 2$ can take place in

the course of elastic scattering with the rate τ_{12}^{-1}. In our model $\tau_{op} \ll \tau_{12}$ and, for strong electron heating, when a considerable fraction of electrons has energies comparable with $\hbar\Omega$, the second subband becomes depleted of electrons. So, in the high-field limit the electron distribution becomes strongly non-equilibrium with almost all electrons occupying the ground subband (even if their energy exceeds the edges of excited subbands). As a result, in this limit the formulae of the previous sections can be applied to the multi-subband case as well.

Now we consider the case of lower electric field and compare the conductivities of two wires having the same electron concentration n but different (either one or two) occupied subbands. If relaxation times (and, hence, electron mobilities) do not depend on p and are the same in the both subbands, the low-field (linear) conductivities of two wires must be equal. So, the CVC for the multi-subband case having the same low- and high-field asymptotics as in the wire with one subband, does not noticeably differ from (13).

Energy dispersion of relaxation times changes the situation. It can be easily seen that in the multi-subband case the average equilibrium electron momentum will be less than in the wire with one subband and the same electron concentration. As a result, for the momentum relaxation time increasing with p, as in section 4, the multi-subband case is characterized by smaller low-field conductivity. The superlinear section of CVC shown in figure 1 will be apparently less pronounced than in the one-subband case. This superlinearity is caused by the increase of electron momentum p and, hence, of τ_i with the electric field. We may expect that in moderate fields, when the optical phonon emission is still of minor importance, relative occupation of excited subbands will increase with F, resulting in a slower increase of effective p.

To summarize, in quantum wires with several occupied subbands the CVC qualitatively has the same properties as described in sections 3 and 4 but quantitative distinctions are possible, especially at not very high fields.

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